Data-driven Geometric Gait Analysis

Ross L. Hatton, Oregon State University
Brian Bittner, University of Michigan
Suresh Ramasamy, Oregon State University
Shai Revzen, University of Michigan

Lie brackets on nonlinear systems—which capture the curvature of their dynamics—describe the net effects of oscillatory inputs [1, 2]. This principle is especially useful for understanding and controlling the motion of locomoting systems—e.g., swimmers or crawlers—whose interaction with the environment is distributed over the body rather than being concentrated at a well-anchored foot. In recent years, we (Hatton and collaborators) have built this Lie bracket principle into a gait analysis paradigm [3–7] that allows us to reason about optimal gait patterns in terms of geometric characteristics such as area and length, as illustrated in Fig. 1.

Applying these principles to systems that lack an analytical model remains an open area of investigation. Although we have had been successful in using exhaustively exploring system dynamics to generate geometric models for simple empirically-observed systems [8, 9] this approach becomes infeasible for high-dimensional systems, or when considering an animal whose motions we cannot directly command.

To fill this gap, we are now combining the geometric paradigm with our (Revzen and collaborators) data-driven modeling toolset [10]. These tools leverage Floquet analysis [11] to extract oscillator dynamics (in particular, their limit cycles) from observed data, and thus identify a motion that the observed system is attempting to track [12–16]. In our new work, we are using the "noise" in the Floquet model (from when the oscillator is away from its limit cycle) to construct a "fat path" model of the system dynamics in the vicinity of the gait, as illustrated in Fig. 2.

This combined model leverages the deep insight provided by the geometric gait analysis paradigm together with the data-driven Floquet model's ability to extract meaning from noisy experimental data. The fat path contains specifically the information required for our geometric tools to determine a gradient of optimality for the gait cycle. Additionally, the geometric structures inform the data-driven tools that certain components of the system model are irrelevant to gait optimality and do not need to be computed.

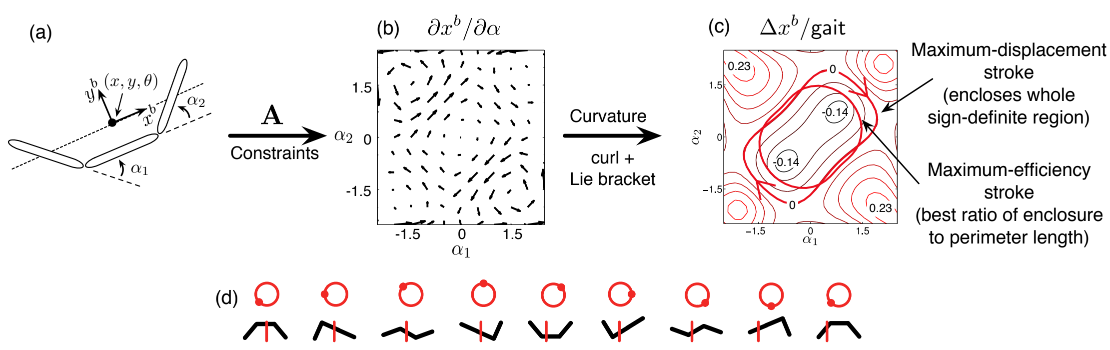

Figure 1: Key elements of our geometric paradigm. Gait displacement depends on how much curvature of the constraints the gait encompasses, and the time-effort cost of executing this gait is the length of the path it traces out in the shape space.

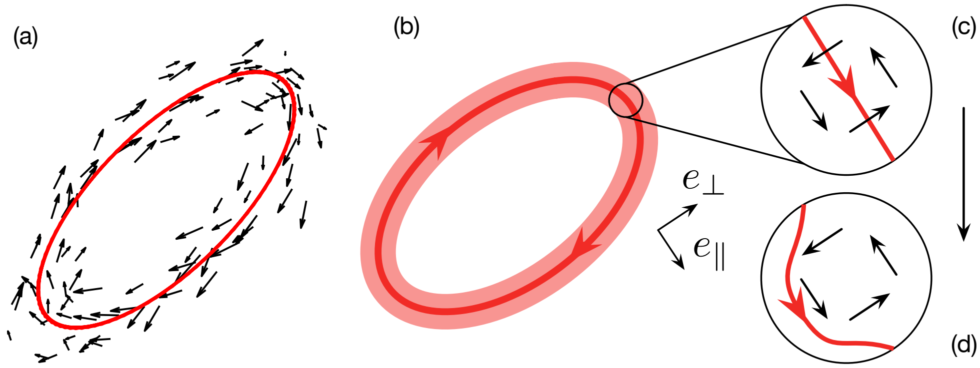

Figure 2: Our data-driven geometric paradigm takes uses the distribution of data samples around the gait cycle identified via Floquet theory (a) to construct a "fat path" model of the system dynamics in the vicinity of the gait (b). At each point on the cycle, we can use the curvature (generalized curl; Lie bracket) to assess the effect that perturbing the gait will have on the net displacement it induces. Here (c) the curvature is in the direction opposite to the gait cycle, so the net displacement can be increased by drawing the gait cycle inward along the $e_\perp$ direction.

References

[1] R. M. Murray and S. S. Sastry, "Nonholonomic motion planning: Steering using sinusoids," *IEEE Transactions on Automatic Control*, vol. 38, no. 5, pp. 700–716, Jan 1993.

[2] K. A. Morgansen, B. I. Triplett, and D. J. Klein, "Geometric methods for modeling and control of free-swimming fin-actuated underwater vehicles," *IEEE Transactions on Robotics*, vol. 23, no. 6, pp. 1184–1199, Jan 2007. [Online]. Available: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=4399955

[3] R. L. Hatton and H. Choset, "Geometric swimming at low and high reynolds numbers," *IEEE Transactions on Robotics*, vol. 29, no. 3, pp. 615–624, June 2013. [Online]. Available: http://ieeexplore.ieee.org/xpls/abs_all.jsp?arnumber=6487415

[4] ——, "Nonconservativity and noncommutativity in locomotion," *European Physical Journal Special Topics: Dynamics of Animal Systems*, vol. 224, no. 17–18, pp. 3141–3174, 12 2015. [Online]. Available: http://epjst.epj.org/articles/epjst/abs/2015/16/epjst150085/epjst150085.html

[5] R. L. Hatton, T. Dear, and H. Choset, "Kinematic cartography and the efficiency of viscous swimming," *IEEE Transactions on Robotics (in press)*, 2017.

[6] S. Ramasamy and R. L. Hatton, "Soap-bubble optimization of gaits," in *The Proceedings of the IEEE Conference on Decision and Control*, Las Vegas, NV, December 2016.

[7] ——, "Geometric gait optimization beyond two dimensions," in *American Control Conference (accepted)*, May 2017.

[8] R. L. Hatton, H. Choset, Y. Ding, and D. I. Goldman, "Geometric visualization of self-propulsion in a complex medium," *Physical Review Letters*, vol. 110, p. 078101, February 2013. [Online]. Available: http://journals.aps.org/prl/abstract/10.1103/PhysRevLett.110.078101

[9] J. Dai, H. Faraji, C. Gong, R. L. Hatton, D. I. Goldman, and H. Choset, "Geometric swimming on a granular surface," in *Proceedings of the Robotics: Science and Systems Conference*, Ann Arbor, Michigan, July 2016. [Online]. Available: www.roboticsproceedings.org/rss12/p12.html

[10] S. Revzen and M. Kvalheim, "Data driven models of legged locomotion," in *SPIE Defense+ Security*. International Society for Optics and Photonics, 2015, pp. 94 671V–94 671V.

[11] J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*. New York: Springer-Verlag, 1983.

[12] H. M. Maus, S. Revzen, J. M. Guckenheimer, C. Ludwig, J. Reger, and A. Seyfarth, "Constructing predictive models of human running," *Journal of the Royal Society Interface*, vol. 12, 2014.

[13] S. Revzen and J. M. Guckenheimer, "Estimating the phase of synchronized oscillators," *Phys Rev E*, vol. 78, no. 5, p. 051907, Nov. 2008.

[14] ——, "Finding the dimension of slow dynamics in a rhythmic system," *J R Soc Lond Interface*, vol. 9, no. 70, pp. 957–971, May 2012.

[15] S. Revzen, S. A. Burden, T. Y. Moore, J.-M. Mongeau, and R. J. Full, "Instantaneous kinematic phase reflects neuromechanical response to lateral perturbations of running cockroaches," *Biol Cybern*, vol. 107, no. 2, pp. 179–200, 2013.

[16] S. Wilshin and S. Revzen, "Phase driven models of unperturbed locomotion," in *Society Integrative and Comparative Biology*, vol. 54. Oxford Univ Press, 2014, p. E226.